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Consistency Checks of Grand Unified Theories

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Abstract

The world averaged values of the electroweak and strong couplings and the lower limits on the proton lifetime are used for consistency checks of Grand Unified Theories (GUT). It is confirmed that new physics outside the Standard Model (SM) is required to obtain unification of the electroweak and strong forces. Such new physics could come from the minimal supersymmetric extension of the SM, which provides unification consistent with the present limits on the proton lifetime, but non-supersymmetric models based on additional split multiplets show similar unification properties. The unification condition strongly restricts the number of GUT's, but cannot distinguish between them. Only future experiments will be able to do so.

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1. Introduction

There has recently been renewed interest in unification of the strong and electroweak interactions based on improved measurements of the couplings associated with the gauge groups $SU(3) \times SU(2) \times U(1)$ of the standard model (SM). In particular, the impressive data which have emerged from the LEP machine at CERN with electron-positron collisions at center-of-mass energy around the Z^0 mass means that one can extract values of the $\alpha_i(M_Z)$ $(i = 1, 2, 3)$ which have unprecedented accuracy. If one then assumes that these three couplings evolve toward higher mass or energy scales, μ, such that they meet at a single unification point in the α_i^{-1}—μ plane, this new accuracy could lead to constraints on the possible new physics which may appear at scales between M_Z and the unification scale, M_{GUT}.

It has been known for several years[1] that with no new physics in the "desert", the unification of the $\alpha_i^{-1}(\mu)$ fails by 2 standard deviations. With the data obtained at LEP by the DELPHI collaboration, this failure was shown to be overwhelming[2]. This is demonstrated in Fig. 1, which shows the evolution of the three couplings using the world averages of 1991 (mainly determined by the LEP data). According to the two-loop renormalization group analysis a single unification point is now excluded by more than 8 standard deviations. Threshold effects near the unification scale do not significantly affect this conclusion.

On the contrary, the minimal supersymmetric extension of the Standard Model (MSSM)[3] gives unification — a result which is by now well known[2,4,5] and which many people consider the first hint, albeit indirect, of supersymmetry in nature. The fit of Ref. [2] gave a predicted scale for the superpartners of $M_{SUSY} = 10^{3\pm1}$GeV where the error has the usual meaning of a 68% confidence interval. It also leads to a unification scale $M_{GUT} \sim 10^{16}$GeV and a proton lifetime safely above the experimental lower limit of a few times 10^{32}y when gauge boson mediated processes are considered. Contributions due to the exchange of Higgs superfields cannot be estimated without further assumptions.

It should be noted that unification is not the motivation for SUSY theories. SUSY was invented some 20 years ago[6] — long before the data was precise enough to test the unification features. It subsequently emerged that gravity could be included in a more natural way[7] and that it helped to solve the hierarchy problem[8], i.e. the fact that the radiative corrections to the Higgs mass squared are of the order of M_{GUT}^2, but the Higgs mass itself is expected to be of the order of the electroweak scale, so large cancellations have to occur. Since bosons and fermions contribute with opposite signs to the self energies, large cancellations automatically occur if the supersymmetric partners have masses of the order of the electroweak scale.

In this paper we first repeat the GUT consistency checks of Ref. [2] using the latest world average values of the couplings. We find that the MSSM unification results are practically unchanged. Then we address the natural question: is such a unification unique to the MSSM case? To answer this question, we discuss a set of non-SUSY models that are equally well consistent with unification. These split multiplet models are of the type originally suggested in Ref. [9] where the motivation was to rescue SU(5) from reports of its demise due mainly to the prediction of too rapid proton decay and of slightly too small a value of the electroweak mixing angle θ_W.

In section 2 of this paper, we give the current world average values for the electroweak and strong couplings and describe the two-loop evolution of these couplings to higher energy scales. In section 3, we first reexamine the consistency of minimal SU(5) and MSSM unification. Then, as an alternative, we consider the question of unification within the context of the non-SUSY split multiplet models. The conclusions are presented in section 4.

2. Evolution of the couplings

In the unified $SU(2) \times U(1)$ theory, the following well known tree-level relations hold between the couplings and the gauge boson masses

$$\begin{array}{rcl} e & = & \sqrt{4\pi\alpha} = g \sin\theta_W = g' \cos\theta_W \\ M_{\mathrm{W}} & = & \frac{1}{2} v g \\ M_{\mathrm{Z}} & = & \frac{1}{2} v \sqrt{g'^2 + g^2} \end{array} \tag{2.1}$$

from which it follows that

$$\sin^2\theta_W = \frac{e^2}{g^2} = \frac{g'^2}{g'^2 + g^2} = 1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2} \tag{2.2}$$

Here g and g' are the couplings of the groups SU(2) and U(1) respectively, α is the fine structure constant and v is the vacuum expectation value of the Higgs field. If the model contains Higgs representations other than doublets, the theory has an additional degree of freedom, usually parametrized by the ρ parameter.

In the SM based on the group $SU(3) \times SU(2) \times U(1)$ we use the usual definitions of the couplings

$$\begin{array}{rcrcl} \alpha_1 & = & (5/3) g'^2/(4\pi) & = & 5\alpha/(3\cos^2\theta_W) \\ \alpha_2 & = & g^2/(4\pi) & = & \alpha/\sin^2\theta_W \\ \alpha_3 & = & g_s^2/(4\pi) & & \end{array} \tag{2.3}$$

where g_s is the SU(3) coupling. The factor of 5/3 in the definition of α_1 has been included for the proper normalization at the unification point[10]. The couplings, when defined as effective values including loop corrections in the gauge boson propagators, become energy dependent ("running"). A running coupling requires the specification of a renormalization prescription, for which one usually uses the modified minimal subtraction ($\overline{MS}$) scheme[11].

In this scheme the world averaged values of the couplings at the Z^0 energy are

$$\begin{array}{rcl} \alpha^{-1}(M_{\mathrm{Z}}) & = & 127.9 \pm 0.2 \\ \sin^2\theta_{\overline{MS}} & = & 0.2333 \pm 0.0008 \\ \alpha_3 & = & 0.113 \pm 0.005 \end{array} \tag{2.4}$$

The value of $\sin^2\theta_{\overline{MS}}$ has been taken from a detailed analysis of all available data by Langacker and Luo[5] which agrees with the latest analysis of the LEP data[12]. The α_3 value corresponds to the world average given by T. Hebbeker at the 1991 Geneva conference[12]. Although the world average has only a total error of 0.003, we have conservatively taken 0.005 as a 68% C.L. error, which is the smallest systematic error of a single α_3 measurement. Since the value of α_3 has been debated very much, we have repeated some of the analysis for a range of α_3 values. We find that the general properties of unification are not sensitive to the precise value of α_3 although specific predictions do change.

For SUSY models, the dimensional reduction $\overline{DR}$ scheme is a more appropriate renormalization scheme[13]. This scheme also has the advantage that all thresholds can be treated by simple step approximations. Thus unification occurs in the $\overline{DR}$ scheme if all three $\alpha_i^{-1}(\mu)$ meet exactly at a point. This crossing point then gives the mass of the heavy gauge bosons. The $\overline{MS}$ and $\overline{DR}$ couplings differ by a small offset

$$\frac{1}{\alpha_i^{\overline{DR}}} = \frac{1}{\alpha_i^{\overline{MS}}} - \frac{C_i}{12\pi} \tag{2.5}$$

where the C_i are the quadratic Casimir coefficients of the group ($C_i = N$ for SU(N) and 0 for U(1) so α_1 stays the same). Throughout the following, we use the $\overline{DR}$ scheme for the MSSM.

The energy dependence of the couplings is completely determined by the particle content and their couplings inside the loop diagrams of the gauge bosons as expressed by the renormalization group (RG) equations. The two-loop RG equations are given by

$$\mu\frac{d}{d\mu}\alpha_i(\mu) = \frac{1}{2\pi}\left(b_i + \sum_{j=1}^{3}\frac{b_{ij}}{4\pi}\alpha_j(\mu)\right)\alpha_i^2(\mu) \tag{2.6}$$

where μ is the energy at which the couplings are evaluated.

For the SM the first order coefficients are[3,14]

$$b_i = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_{\text{Fam}}\begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + N_{\text{Higgs}}\begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} \tag{2.7}$$

while for the MSSM they have been calculated to be[14]

$$b_i = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{\text{Fam}}\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{\text{Higgs}}\begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} \tag{2.8}$$

where N_{Fam} is the number of families of matter (super)multiplets and N_{Higgs} is the number of Higgs doublets. $N_{\text{Fam}} = 3$ and $N_{\text{Higgs}} = 1$ or 2 for the SM or MSSM, respectively. Note that in the MSSM the dominating first order coefficients lead to a weaker running of α_3 than predicted by the SM, while the running of α_2 has the opposite sign and α_1 runs somewhat faster.

Neglecting Yukawa couplings, the second-order coefficients for the SM are[14]

$$b_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{136}{3} & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_{\rm Fam} \begin{pmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ \frac{1}{5} & \frac{49}{3} & 4 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} + N_{\rm Higgs} \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.9}$$

For the MSSM they become[14]

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_{\rm Fam} \begin{pmatrix} \frac{38}{15} & \frac{6}{5} & \frac{88}{15} \\ \frac{2}{5} & 14 & 8 \\ \frac{11}{15} & 3 & \frac{68}{3} \end{pmatrix} + N_{\rm Higgs} \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.10}$$

In addition to the SM and the MSSM, we are interested in the β-function coefficients for the split multiplet models of Ref. [9]. The idea behind these models is to add split fermion multiplets $(5 + \overline{5})$ and $(10 + \overline{10})$ to the minimal three families $3\,(\overline{5} + 10)$ in SU(5). In the present paper we slightly generalize this to include fermions and scalars.

The content of the $(5+\overline{5})$ and $(10+\overline{10})$ real representations of SU(5) decomposes under $SU(5) \supset SU(3) \times SU(2) \times U(1)$ as follows[9]

$$\begin{array}{rcl} A & = & (1,2)_{-1} + (1,2)_1 \\ B & = & (\overline{3},1)_{2/3} + (3,1)_{-2/3} \\ C & = & (3,2)_{1/3} + (\overline{3},2)_{-1/3} \\ D & = & (\overline{3},1)_{-4/3} + (3,1)_{4/3} \\ E & = & (1,1)_2 + (1,1)_{-2} \end{array} \tag{2.11}$$

where the first number in brackets indicates the SU(3) color charge (triplet or singlet), the second one the SU(2) weak isospin T (doublet or singlet), while the subscript denotes the hypercharge Y, which is related to the electric charge Q by $\mathrm{Q} = \mathrm{T}_3 + \frac{1}{2}\mathrm{Y}$.

We recall that, in minimal SU(5), a usual quark-lepton family fills out a complex representation $(\overline{5} + 10)$ where the 5 contains (for the first family) the chiral neutrino-electron $(\nu_e, e^-)_{\rm L}$ doublet (A) and the anti-down quark $\overline{d}_{{\rm L}\alpha}$ (B), while the $\overline{10}$ contains the up-down $(u_\alpha, d_\alpha)_{\rm L}$ doublet (C), the anti-up $\overline{u}_{{\rm L}\alpha}$ (D) and the positron $e^+_{\rm L}$ (E). In (2.11), however, it is important to emphasize that the content of the real representations A through E are non-chiral so that identification with the pieces of a usual chiral family, while a useful mnemonic, is not exact because, unlike the usual chiral quarks and leptons, these real representations can acquire gauge-invariant Dirac masses without breaking the $SU(2) \times U(1)$ electroweak symmetry.

The additional contributions to the one-loop β-functions from A through E are given by (for one scalar)

$$\begin{array}{ccccc} \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} & \mathrm{E} \\ \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{15} \\ 0 \\ \frac{1}{6} \end{pmatrix} & \begin{pmatrix} \frac{1}{30} \\ \frac{1}{2} \\ \frac{1}{3} \end{pmatrix} & \begin{pmatrix} \frac{4}{15} \\ 0 \\ \frac{1}{6} \end{pmatrix} & \begin{pmatrix} \frac{1}{5} \\ 0 \\ 0 \end{pmatrix} \end{array} \tag{2.12}$$

	Fermion	Scalar
A	$\begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{49}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix}$
B	$\begin{pmatrix} \frac{4}{75} & 0 & \frac{16}{15} \\ 0 & 0 & 0 \\ \frac{2}{15} & 0 & \frac{38}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{4}{75} & 0 & \frac{16}{15} \\ 0 & 0 & 0 \\ \frac{2}{15} & 0 & \frac{11}{3} \end{pmatrix}$
C	$\begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} \\ \frac{1}{10} & \frac{49}{2} & 8 \\ \frac{1}{15} & 3 & \frac{76}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} \\ \frac{1}{10} & \frac{13}{2} & 8 \\ \frac{1}{15} & 3 & \frac{22}{3} \end{pmatrix}$
D	$\begin{pmatrix} \frac{64}{75} & 0 & \frac{64}{15} \\ 0 & 0 & 0 \\ \frac{8}{15} & 0 & \frac{38}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{64}{75} & 0 & \frac{64}{15} \\ 0 & 0 & 0 \\ \frac{8}{15} & 0 & \frac{11}{3} \end{pmatrix}$
E	$\begin{pmatrix} \frac{36}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{36}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Table 1: Contributions to the two-loop β-functions from additional A through E fermions and scalars.

At one-loop, one fermion counts 4 times as much as a scalar. However, at two-loops, this is no longer the case. The relevant changes in the second order coefficients (Δb_i) are given in Table 1.

In following the evolution in the MSSM and split multiplet models, we assume that all new particles have an intermediate mass scale, which we call $M_{\rm SUSY}$ and $M_{\rm threshold}$, respectively. Thus for smaller energies, the RG equations are determined by the SM β-function coefficients, (2.7) and (2.9). The new particles only enter into the RG equations above $M_{\rm SUSY}$ or $M_{\rm threshold}$. For the split multiplets, we assume that all new particles have masses exactly equal to $M_{\rm threshold}$ for simplicity in treating the threshold. Even if these particles are not degenerate, $M_{\rm threshold}$ or $M_{\rm SUSY}$ can still be viewed as an effective mass scale for these particles.

In addition, the Higgs Yukawa couplings enter the RG equations at two-loop order. However, we have not included them because they depend on unknown parameters like the top quark mass and, for MSSM, the ratio of the Higgs vacuum expectation values. Since the two-loop evolution of the β-functions is only slightly changed from the one-loop evolution, we believe neglecting the Yukawa couplings will change little.

The RG equations can be rewritten as

$$\frac{d}{d\ln\mu}\alpha_i^{-1}(\mu) = \frac{-1}{2\pi}\left(b_i + \sum_{j=1}^{3}\frac{b_{ij}}{4\pi}\alpha_j(\mu) + O(\alpha_j^2)\right) \tag{2.13}$$

and we see that in first order the equations for the three α_i^{-1} are independent with a linear solution in the α_i^{-1}—$\log\mu$ plane. When the second order contributions are taken into account, the equations become coupled and the running of each α_i^{-1} depends on the values of the other two couplings. However, the second order effects are small because of the additional factor $\alpha_j/(4\pi) \leq 0.01$. Higher orders are presumably even smaller by additional powers of $\alpha_j/(4\pi)$. We solve (2.13) by numerical integration.

3. Consistency of the models with the GUT hypothesis

For unification in the $\overline{DR}$ scheme, all three couplings $\alpha_i^{-1}(\mu)$ must cross at a single unification point in the α_i^{-1}—μ plane given by M_{GUT} and $\alpha_{\mathrm{GUT}}^{-1}$ (the inverse of the unified coupling). Thus in these models, as done for the first time in Ref. [2], we fit for the intermediate scale, M_{SUSY} or $M_{\mathrm{threshold}}$, as well as M_{GUT} and $\alpha_{\mathrm{GUT}}^{-1}$ by minimizing χ^2 which is given by

$$\chi^2 = \sum_{i=1}^{3} \frac{(\alpha_i^{-1}(\mu) - \alpha_{\mathrm{GUT}}^{-1})}{\sigma_i^2} \tag{3.1}$$

where σ_i is the error on α_i^{-1}.

Since there are as many parameters in this fit as there are couplings, a perfect fit with $\chi^2 = 0$ is usually possible. However, because the intermediate scale, M_{SUSY} (or $M_{\mathrm{threshold}}$), must fall between M_{Z} and M_{GUT}, this is not always the case. A perfect fit corresponds to M_{SUSY} (or $M_{\mathrm{threshold}}$) between these two limits. In other cases, one could only obtain a best fit ($\chi^2 > 0$) at one of the end points M_{SUSY} (or $M_{\mathrm{threshold}}$) $= M_{\mathrm{Z}}$ or M_{GUT}. In addition, we demand that M_{GUT} has to be larger than 2×10^{15}GeV in order to be consistent with proton lifetime limits, a non-trivial constraint.

3.1. Minimal SU(5)

The simplest Grand Unified Theory is the minimal SU(5) model with three families of matter and one Higgs doublet. In this model, one family of quarks and leptons fits into the $\bar{5}$ and 10 representation of SU(5). In addition to the 12 known gauge bosons, there are 12 new gauge bosons which can induce transitions between quarks and leptons.

The evolutions of the three couplings are shown in Fig. 1 for the minimal SM. It is clear that a single unification point can not be obtained within minimal SU(5) with the present errors (indicated by the width of the lines). The α_3 coupling misses the crossing point of the other two by more than 8 standard deviations. Threshold effects near the grand unification scale and higher order contributions do not modify this conclusion.

3.2. Minimal Supersymmetric Standard Model

Fig. 2 shows that, within the MSSM, unification is obtained. In the fit which minimizes the differences between the couplings at the unification point M_{GUT}, both M_{SUSY} and M_{GUT} are free parameters.

From the fit, one finds (in perfect agreement with Ref. [2])

$$\begin{array}{rcl} M_{\mathrm{SUSY}} & = & 10^{3.4\pm0.9\pm0.4}\mathrm{GeV} \\ M_{\mathrm{GUT}} & = & 10^{15.8\pm0.3\pm0.1}\mathrm{GeV} \\ \alpha_{\mathrm{GUT}}^{-1} & = & 26.3\pm1.9\pm1.0 \end{array} \tag{3.2}$$

The first errors in these values originate from the experimental uncertainties in the couplings (mainly from the error of $\alpha_3(M_Z)$), while the second error is an estimate from the uncertainty in the SUSY mass spectrum. For this estimate we assumed the strongly interacting sparticles to all have the mass $M_{\tilde{q},\tilde{g}}$ and the non-strongly interacting sparticles all to have the mass $M_{\tilde{l},\tilde{\mathrm{W}},\tilde{\mathrm{Z}},\tilde{\gamma}}$ and varied the ratio of these masses between 1 and 4, which is a reasonable range within the MSSM[16]. For the values in Eqn. (3.2), we have assumed this ratio to be 2. From these values one observes that the uncertainties from the light thresholds are not large compared with the experimental errors. Concerning the heavy thresholds, we assumed that any new heavy gauge or Higgs bosons are degenerate with M_{GUT}, in which case they do not influence the running of the couplings. From proton decay limits they indeed have to be close to M_{GUT}[17]. If one allows them to have masses far below M_{GUT} with large uncertainties, one can not determine M_{SUSY} anymore, even with a perfect knowledge of the couplings[18]. One Higgs doublet was assumed to have a mass near M_Z, while the mass of a second Higgs doublet (in the MSSM) was varied between M_Z and several times M_{SUSY}. The effects of such variations is small compared with the experimental errors. Fig. 3 demonstrates the sensitivity of M_{SUSY} to the value of the strong coupling for different assumptions on the sparticle mass splittings.

One can ask: what is the meaning of the parameter M_{SUSY}? Far above and below the threshold for the SUSY particles, the slopes of the inverse couplings are well known. Extrapolating them linearly into the narrow threshold region defines an effective mass scale M_{SUSY}, defined as the energy where the SM and MSSM slopes cross. Clearly, a single parameter is inadequate to parametrize the SUSY mass spectrum, as emphasized in Ref. [17]; to do so one needs a minimum of 5 parameters. However, from the unification of the three couplings, one can not determine so many parameters. One knows that within the MSSM the spread in sparticle masses is small compared with M_{GUT}[16]. Therefore, M_{SUSY}, being some effective mass in this rather narrow threshold region, is the best estimate of the sparticle masses we have. Unfortunately, even this single parameter has large errors, since M_{SUSY} enters only logarithmically into the extrapolation of the couplings. The 68% C.L. error already spans two orders of magnitude, so the 95% C.L. error spans four orders of magnitude, i.e. $10^{1.4} < M_{\mathrm{SUSY}} < 10^{5.4}$GeV, a result agreeing with our previous results[2] and obtained later also in Ref. [19]. Nevertheless, one knows that within the MSSM at least one of the Higgs particles will have a mass below 170 GeV[20], which should certainly be within reach of the next generation of accelerators.

3.3. Split multiplets models

In the non-SUSY case, we have searched over all split combinations of A through E allowing up to a total of five additional representations. In addition, we took no more than two fermions or three scalars of a particular type. These restrictions are unimportant and are motivated by the desire to introduce a minimum of additional states. An extension of the search would only lead to even more consistent unification models.

For each choice of model, we further limit $M_{\text{threshold}}$ to vary between M_Z and 10 TeV which we chose arbitrarily to limit the possible new physics to an experimentally interesting regime. In principle, the upper limit for $M_{\text{threshold}}$ is the unification scale, M_{GUT}. A list of all resulting models with a maximum of four additional representations and with $\chi^2 = 0$ and $M_{\text{GUT}} > 2 \times 10^{15}$GeV is shown in Table 2. Without the constraint $M_{\text{GUT}} > 2 \times 10^{15}$GeV other split models are possible, as shown e.g. in Ref.[21].

Fermions A B C D E	Scalars A B C D E	# irreps. beyond SM	$M_{\text{threshold}}$ (GeV)	M_{GUT} (GeV)	α_{GUT}^{-1}
0 1 1 0 0	1 0 0 0 0	3	1600	9.4×10^{15}	35.2
0 0 1 0 0	0 0 0 2 0	3	210	2.4×10^{15}	35.0
0 0 1 0 0	0 1 0 1 0	3	5500	3.0×10^{15}	36.2
0 0 1 0 0	0 1 0 1 1	4	260	2.5×10^{15}	35.1
0 1 1 0 0	0 0 1 1 0	4	470	3.6×10^{16}	32.9
0 0 1 1 0	1 0 1 0 0	4	740	2.6×10^{15}	32.1
0 0 1 0 0	0 3 0 0 0	4	1300	9.4×10^{15}	35.9
0 0 1 0 0	0 2 0 0 1	4	6400	3.0×10^{15}	36.2

Table 2: A complete list of models with up to four additional irreducible representations which exhibit perfect unification.

Of these, perhaps the most economical is the one with one A scalar (corresponding to a second Higgs doublet) and fermions in the B and C representations of Eqn. (2.11). We call this the ABC model.

The α_i^{-1}—μ plot for the ABC model is shown in Fig. 4. The values of the unification parameters are given by

$$\begin{aligned} M_{\text{threshold}} &= 10^{3.2\pm.9}\text{GeV} \\ M_{\text{GUT}} &= 10^{16.0\pm.3}\text{GeV} \\ \alpha_{\text{GUT}}^{-1} &= 35.2 \pm 0.6 \end{aligned} \tag{3.3}$$

The value of M_{GUT} in this model is very similar to the one obtained in the MSSM case. This similarity is due to the fact that at one-loop order, the couplings $\alpha_i^{-1}(\mu)$ evolve linearly, and M_{GUT} depends only on the differences, $b_{1-2} \equiv b_1 - b_2$ and $b_{2-3} \equiv b_2 - b_3$. With respect to the SM, both the MSSM and the ABC model display the same variation of the differences ($\Delta b_{1-2} = -5/3$ and $\Delta b_{2-3} = 1/6$) so a one-loop analysis would give identical values for M_{SUSY} (or $M_{\text{threshold}}$) and M_{GUT}. The slight differences come from the two-loop corrections. We conclude that these values of $\Delta b_{1-2} = -5/3$ and $\Delta b_{2-3} = 1/6$ are the interesting ones, if one imposes unification with a breaking scale of the order of 1 TeV.

For the GUT's embedding the MSSM (the ABC) model, the proton lifetime can be estimated by

$$\tau_{\rm p} \approx \frac{1}{\alpha_{\rm GUT}^2} \frac{M_{\rm GUT}^4}{M_{\rm p}^5} \tag{3.4}$$

which corresponds to $10^{34.5\pm1.2}$y ($10^{35.5\pm1.2}$y) for $M_{\rm GUT} = 10^{15.8\pm0.3}$GeV ($M_{\rm GUT} = 10^{16.0\pm0.3}$GeV). Eqn. 3.4 is valid in the MSSM only if non-gauge contributions are neglected.

Any of the models listed in Table 2 will give a unification plot of α_i^{-1}—μ as impressive as Figs. 2 and 4. Note that the lesser number of states make α_{GUT} smaller than in the MSSM.

4. Summary

It is irresistible to use the new precise values of the three standard couplings to study unification. Together with the lower limits on the proton lifetime they allow stringent consistency checks of the various models.

In essence, unification of the strong- and electroweak forces fails with no new physics beyond $M_{\rm Z}$, since within the SM the three couplings never become equal at a single energy.

As is well known, the minimal supersymmetric extension of the SM yields an amazingly consistent unification picture, but non-supersymmetric models with similar unification properties have been found too.

In these models, additional multiplets of quarks and leptons are introduced, which are split between the TeV energy scale and the unification scale. The advantage of such models is that one needs fewer new particles than in the MSSM. However, the MSSM possesses a new symmetry of greater elegance and motivation than these split multiplet models.

From the unification properties, one cannot distinguish between these models; hence one cannot predict the nature of the new physics. Only consistency checks of the various models can be made, and nothing more, as stressed in Ref. [2]. Experiments at future accelerators and the possible observation of proton decay will tell whether the forces are unified and which new physics beyond the SM is required to achieve this.

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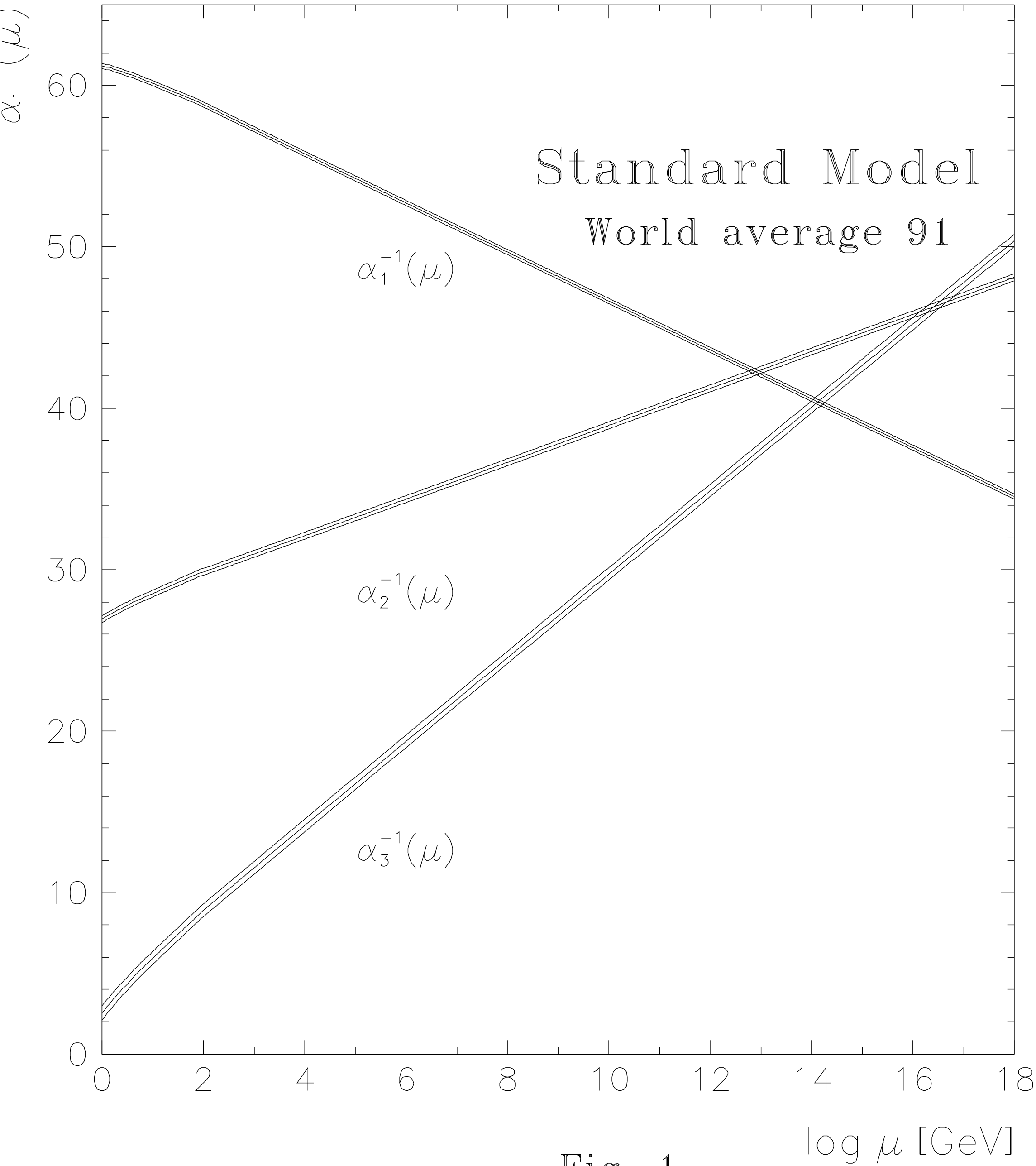
Figure Captions

Fig. 1. Evolution of the inverse of the three couplings in the minimal SM.

Fig. 2. Evolution of the inverse of the three couplings in the MSSM. To guide the eyes the lines above $M_{\rm GUT}$ follow the prediction from the supersymmetric SU(5) model.

Fig. 3. The $M_{\rm SUSY}$ (a) and $M_{\rm GUT}$ (b) energy scales are shown as a function of $\alpha_3(M_{\rm Z})$. The uncertainty in $M_{\rm GUT}$ and $M_{\rm SUSY}$ from the errors in $\alpha_1(M_{\rm Z})$ and $\alpha_2(M_{\rm Z})$ are indicated by the dashed area. The dashed lines assume all sparticles to be degenerate, while the solid (dotted) lines assume the strongly interacting sparticles to be two (four) times heavier than the others.

Fig. 4. Evolution of the inverse couplings in the ABC model.



Standard Model
World average 91
$\alpha_i^{-1}(\mu)$
$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
0
10
20
30
40
50
60
0
2
4
6
8
10
12
14
16
18
log μ [GeV]

Fig. 1

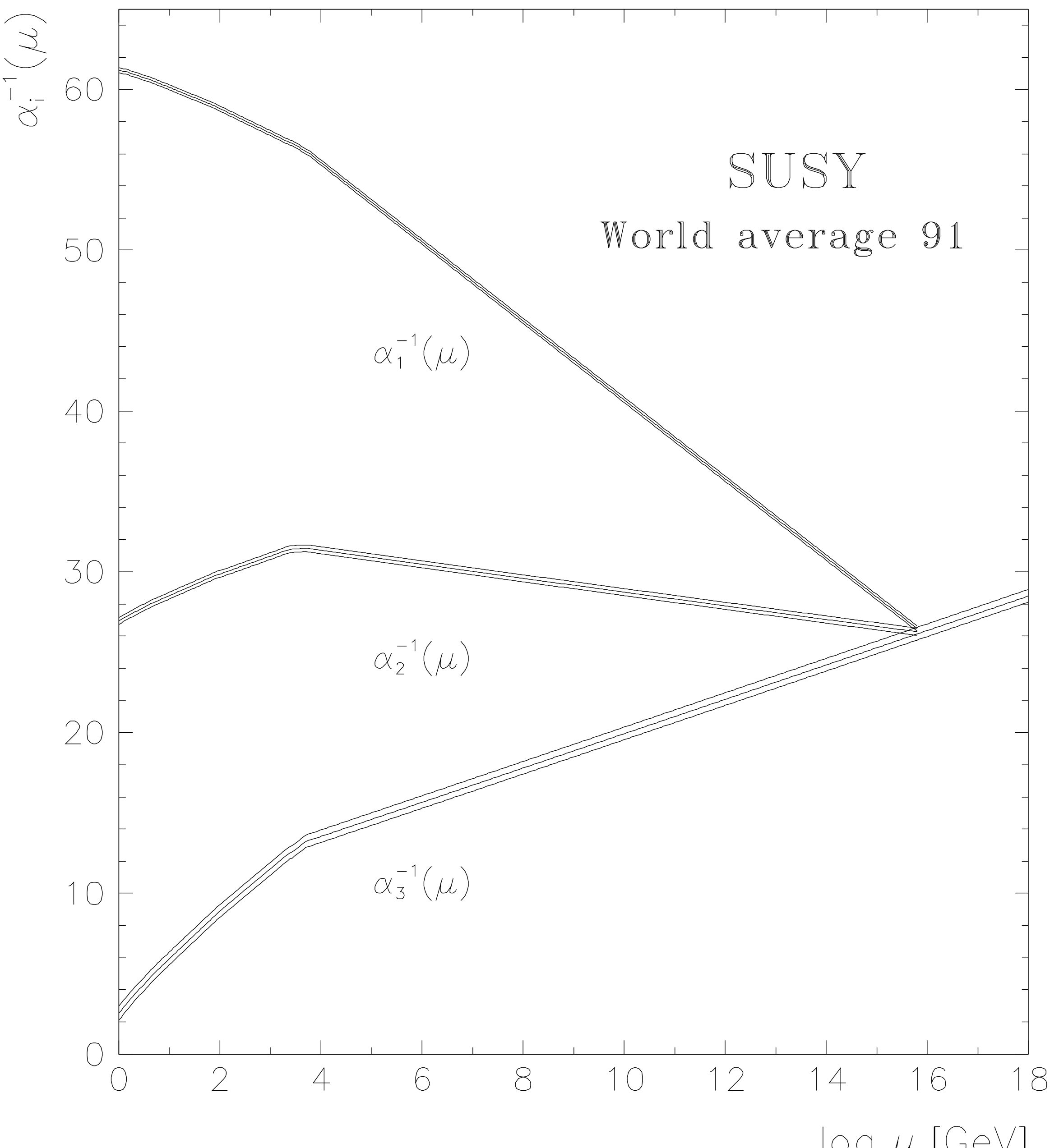
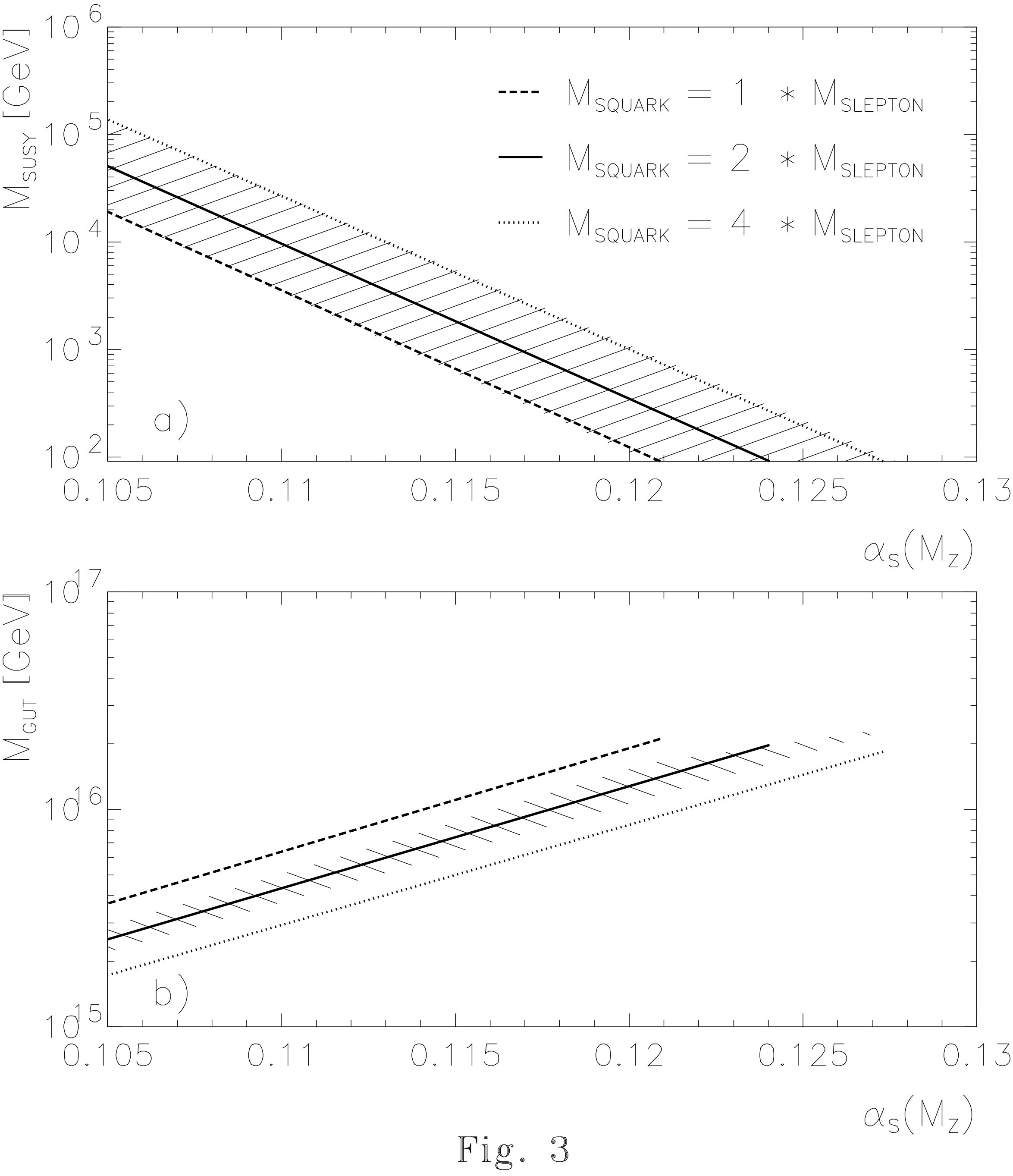

SUSY
World average 91
$\alpha_i^{-1}(\mu)$
$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
0
10
20
30
40
50
60
0
2
4
6
8
10
12
14
16
18
log μ [GeV]

Fig. 2



10^6
10^5
10^4
10^3
10^2
M_{SUSY} [GeV]
$M_{SQUARK} = 1 * M_{SLEPTON}$
$M_{SQUARK} = 2 * M_{SLEPTON}$
$M_{SQUARK} = 4 * M_{SLEPTON}$
a)
0.105
0.11
0.115
0.12
0.125
0.13
$\alpha_s(M_Z)$
10^{17}
10^{16}
10^{15}
M_{GUT} [GeV]
b)
0.105
0.11
0.115
0.12
0.125
0.13
$\alpha_s(M_Z)$

Fig. 3

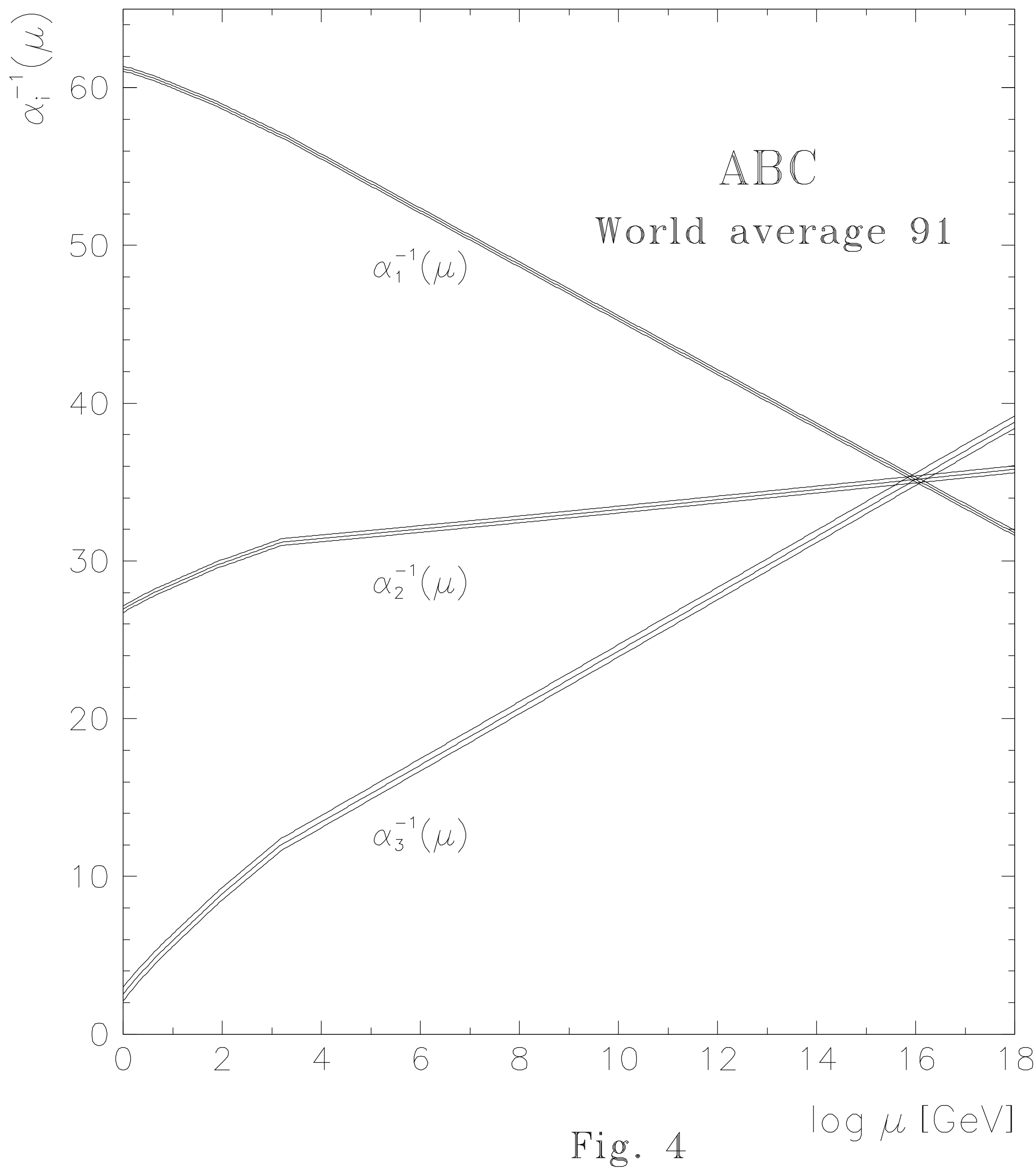

$\alpha_i^{-1}(\mu)$
ABC
World average 91
$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
0
10
20
30
40
50
60
0
2
4
6
8
10
12
14
16
18
log μ [GeV]

Fig. 4